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Study of light kaonic nuclei with a Chiral SU(3) - based $\bar{K}N$ interaction

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Abstract. We have investigated the prototype kaonic nucleus, ppK^-, using the method of Antisymmetrized Molecular Dynamics (AMD). In the present study we use a realistic NN potential with strongly repulsive core, and a Chiral SU(3)-based $\bar{K}N$ interaction which is energy-dependent and includes both s- and p-wave interactions. We find that no self-consistent solutions exist when the range parameter of the $\bar{K}N$ interaction is less than 0.67 fm. Due to the strong repulsion of the NN interaction at short distance, the two nucleons in ppK^- keep a distance of about 1.2 fm and the total binding energy of ppK^- does not exceed about 50 MeV in the scope of our current analysis.

PACS. 13.75.Jz Kaon-baryon interactions – 21.30.Fe Forces in hadronic systems and effective interactions – 21.45.+v Few-body systems

Kaon-nuclear systems, with a K^- tightly bound to a nuclear core, have recently received considerable interest. Previous studies [1,2,3] suggested lots of exotic properties of light kaonic nuclei, as a consequence of the strongly attractive $I = 0$ $\bar{K}N$ interaction. Several experiments have been performed by various groups [4,5,6,7]. Recently the FINUDA group reported measurements [5] which they interpret as signals for the formation of ppK^- clusters, the basic prototypes of kaonic nuclei. While this interpretation is not without controversy, the quest for deeply bound kaon-nuclear states has attracted a great amount of activity in the field.

Our investigations are aimed at a detailed understanding of the conditions under which kaonic nuclei can exist. As a first step we explore the ppK^- system, focusing on the following two points: the role of the repulsive core of the NN interaction and the implementation of a Chiral SU(3)-based $\bar{K}N$ interaction. If the kaonic nuclei are dense and compact as suggested by previous calculations [1,2,3], the strong repulsion of the NN interaction at short distance is expected to be very important in acting against such compression into very dense configurations. The $\bar{K}N$ interaction is a key ingredient in the study of kaonic nuclei. In the present calculations we adopt a theoretically motivated interaction instead of the previous phenomenological one [1]. The s-wave part of this interaction is derived from chiral SU(3) theory [8] and includes the strong energy dependence resulting from coupled-channel dynamics. An energy dependent p-wave interaction dominated by the $\Sigma(1385)$ is also incorporated.

We investigate light kaonic nuclei in a fully microscopic way employing the variational Antisymmetrized Molecular Dynamics (AMD) method. In the AMD framework, single nucleon wave functions $|\varphi_i\rangle$ and the kaon wave function $|\varphi_K\rangle$ are constructed as follows;

$$|\varphi_i\rangle = \sum_{\alpha=1}^{N_n} C_\alpha^i \exp\left[-\nu\left(\boldsymbol{r} - \frac{\boldsymbol{Z}_\alpha^i}{\sqrt{\nu}}\right)^2\right] |\sigma_i\rangle|\tau_\alpha^i\rangle, \quad (1)$$

$$|\varphi_K\rangle = \sum_{\alpha=1}^{N_K} C_\alpha^K \exp\left[-\nu\left(\boldsymbol{r} - \frac{\boldsymbol{Z}_\alpha^K}{\sqrt{\nu}}\right)^2\right] |\tau_\alpha^K\rangle. \quad (2)$$

The trial wave function of each nucleon (or the kaon) is described as a superposition of N_n (N_K) Gaussian wave packets whose centers $\{\boldsymbol{Z}_\alpha^i\}$ ($\{\boldsymbol{Z}_\alpha^K\}$) are different from each other; ν is a width parameter of the Gaussian wave packets; $|\sigma_i\rangle$ represents a spin wave function, $|\uparrow\rangle$ or $|\downarrow\rangle$; $|\tau_\alpha^i\rangle$ and $|\tau_\alpha^K\rangle$ are isospin wave functions and have the following form:

$$|\tau_\alpha^i\rangle = \left(\frac{1}{2} + \gamma_\alpha^i\right)|p\rangle + \left(\frac{1}{2} - \gamma_\alpha^i\right)|n\rangle, \quad (3)$$

$$|\tau_\alpha^K\rangle = \left(\frac{1}{2} + \gamma_\alpha^K\right)|\bar{K}^0\rangle + \left(\frac{1}{2} - \gamma_\alpha^K\right)|K^-\rangle \quad (4)$$

where $\{\gamma_\alpha^i\}$ and $\{\gamma_\alpha^K\}$ are variational parameters.

The total wave function is constructed by antisymmetrizing a set of nucleon wave functions, Eq. (1), and combining it with the kaon wave function, Eq. (2): $|\Phi\rangle = \mathcal{A}[|\varphi_i(a)\rangle] \otimes |\varphi_K\rangle$. This wave function is projected onto an

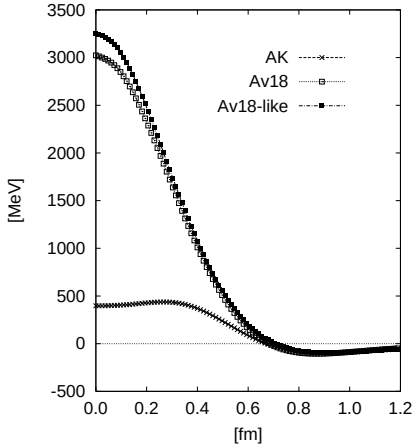


Fig. 1. Comparison of NN potentials in ${}^1S_0$ channel. "AK" is the NN potential used in the previous study. "Av18" is the Argonne v18 potential. "Av18-like" is the NN potential used in the present study.

eigenstate of angular momentum (J), isospin (T), charge (C) and parity (P): $|P_J P_T P_C P_P\ \Phi\rangle$. Given the charge-mixed state, Eqs. (3) and (4), and the charge projection (P_C), we can adequately treat the $K^-p/\bar{K}^0n$ mixing which is caused by the $\bar{K}N$ interaction. All variational parameters $\{C^i_\alpha, \mathbf{Z}^i_\alpha, \gamma^i_\alpha; C^K_\alpha, \mathbf{Z}^K_\alpha,\ \gamma^K_\alpha\}$, which are complex numbers, are determined by the frictional cooling equation. Details of the AMD method are explained in Ref. [3].

The NN interaction used here is guided by the Argonne v18 (Av18) potential [9]. Fig. 1 shows the ${}^1S_0$ channel of the potential used in our study ("Av18-like"), the original Argonne v18 potential ("Av18"), and the potential used in the previous studies ("AK") [1,2,3]. Evidently, the present realistic potential has a strongly repulsive core in contrast to the previously used potential. Since this previous potential has been constructed using the G-matrix method [1], the repulsive-core part is smoothed out.

The s-wave and p-wave $\bar{K}N$ interactions have the following forms:

$$v_{KN,S}(\boldsymbol{r}_N - \boldsymbol{r}_K, \omega) = -2\pi \left(\frac{\omega + M_N}{\omega M_N}\right) F_{KN}(\omega) \times \frac{1}{\pi^{3/2} a_s^3} \exp[-(\boldsymbol{r}_N - \boldsymbol{r}_K)^2/a_s^2], \quad (5)$$

$$v_{KN,P}(\boldsymbol{r}_N - \boldsymbol{r}_K, \omega) = -2\pi \left(\frac{\omega + M_N}{\omega M_N}\right) C_{KN}(\omega) \times \frac{1}{\pi^{3/2} a_p^3} \nabla \exp[-(\boldsymbol{r}_N - \boldsymbol{r}_K)^2/a_p^2] \nabla, \quad (6)$$

where M_N is the nucleon mass and ω is the kaon energy. The present $\bar{K}N$ potentials have a strong energy dependence. The s- and p-wave potentials have Gaussian shapes with range parameters a_s and a_p. $F_{KN}(\omega)$ and $C_{KN}(\omega)$ are the s-wave $\bar{K}N$ scattering amplitude and the energy dependent p-wave $\bar{K}N$ scattering "volume", respectively. The amplitude $F_{KN}(\omega)$ is derived from Chiral SU(3) theory [8]. Its energy dependence reflects primarily the strong $\bar{K}N \leftrightarrow \pi\Sigma$ coupled-channels dynamics which drives the $\Lambda(1405)$ resonance. The $C_{KN}(\omega)$ is an updated version of the parameterization introduced long ago in Ref. [10]. It features the dominance of the $\Sigma(1385)$ in p-wave $I = 1$ channel for which $C_{K^-n} \simeq 2C_{K^-p}$ holds. Fig. 2 shows $F_{KN}(\omega)$ and $C_{KN}(\omega)$.

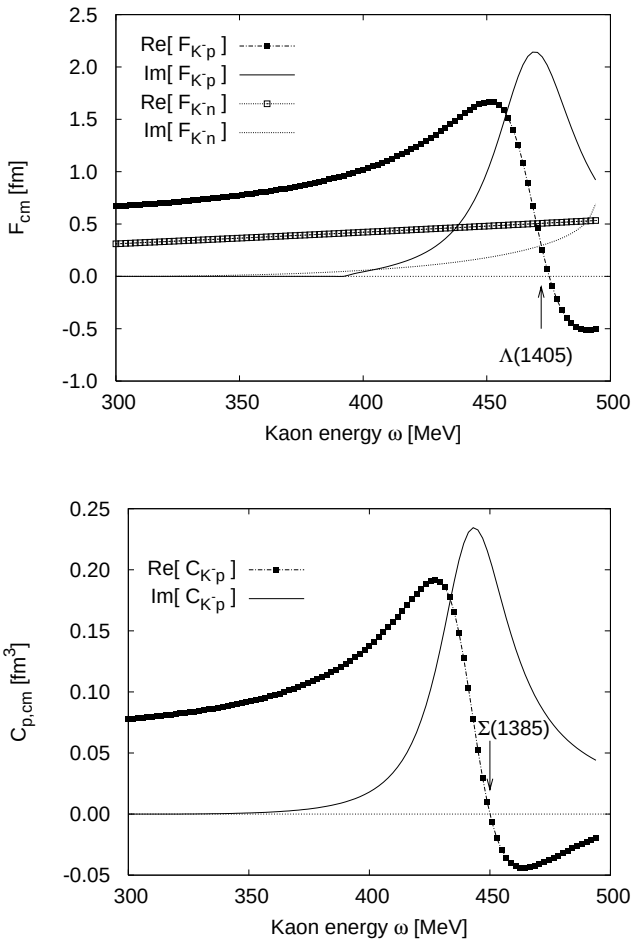


Fig. 2. The s-wave amplitude $F_{KN}(\omega)$ (upper panel) and the p-wave scattering "volume" $C_{KN}(\omega)$ (lower panel).

The procedure of the present variational calculation is as follows. In a first step we treat the p-wave $\bar{K}N$ interaction perturbatively and omit the imaginary parts of the s- and p-wave $\bar{K}N$ interactions. The total Hamiltonian $\hat{H}$ is

$$\hat{H} = \hat{H}_0 + \mathrm{Re}[\hat{V}_{KN,P}(\omega)] \quad (7)$$

$$\hat{H}_0 \equiv \hat{T} + \hat{V}_{NN} + \mathrm{Re}[\hat{V}_{KN,S}(\omega)] + \hat{V}_{Coul.} - \hat{T}_{CM}. \quad (8)$$

Here, $\hat{T}$, $\hat{V}_{NN}$ and $\hat{V}_{Coul.}$ correspond to the total kinetic energy, NN central potential and Coulomb potential, respectively. We use all channels of the Av18-like potential as our $\hat{V}_{NN}$, although channels other than ${}^1S_0$ turn out not to be important in ppK^-. $\hat{V}_{KN,S}(\omega)$ and $\hat{V}_{KN,P}(\omega)$ are the s- and p-wave $\bar{K}N$ potentials of Eqs. (5) and (6), respectively. The center-of-mass motion energy of the whole

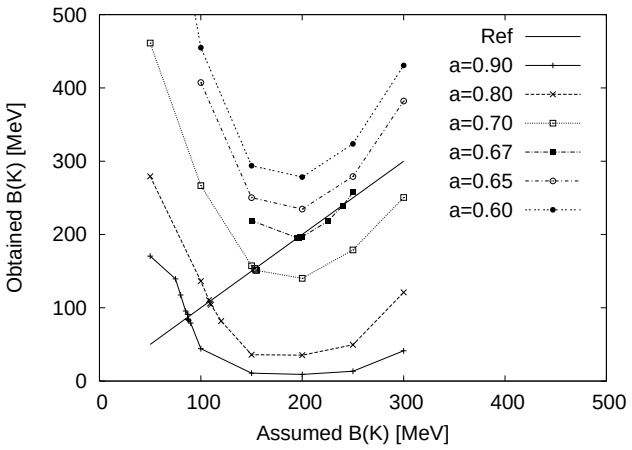


Fig. 3. Self consistency of the kaon's binding energy $B(K)$. The horizontal axis $B(K)_{assumed}$ is the kaon's binding energy assumed as input in order to determined the strength of $\bar{K}N$ interaction. The vertical axis $B(K)_{obtained}$ is the kaon's binding energy calculated with such a $\bar{K}N$ interaction. "Ref" is the line $B(K)_{assumed} = B(K)_{obtained}$. The curves correspond to results calculated with different range parameters a.

system, $\hat{T}_{CM}$, is subtracted. We determine an AMD wave function $|\Phi_0\rangle$ by the energy variation for the Hamiltonian $\hat{H}_0$. Then we calculate the total energy E_{tot} which is the expectation value of the total Hamiltonian $\hat{H}$ with this wave function $|\Phi_0\rangle$. Secondly, we must take into account the self-consistency of the kaon's energy since the present $\bar{K}N$ interaction is strongly energy dependent. We obtain the self-consistent solution by the following steps. 1. We assume a kaon binding energy $B(K)_{assumed}$ as trial input. Once the kaon's energy is determined as $\omega = m_K - B(K)_{assumed}$, the strength of the $\bar{K}N$ interaction is fixed and the Hamiltonian is also determined. 2. We perform the energy variation and treat the p-wave $\bar{K}N$ interaction perturbatively so as to calculate the total energy. We compute separately the kaon binding energy, $B(K)_{obtained}$. 3. We compare $B(K)_{obtained}$ with $B(K)_{assumed}$. If they differ, we return to step 1 and change the input value of $B(K)_{assumed}$. We repeat the cycle 1 to 3 until $B(K)_{obtained}$ coincides with $B(K)_{assumed}$. When $B(K)_{obtained} = B(K)_{assumed}$, a self-consistent solution is found.

We comment briefly on the kaon binding energy $B(K)$. In kaonic nuclei, $B(K)$ is different from the total energy, E_{tot}, because the nuclear components rearrange themselves under the influence of the strong attraction from the kaon field located at the center of the nucleus. In fact, the nuclear part in a kaonic nucleus is in an excited state relative to the original nuclear system. We define $B(K) = -(E_{tot} - E_{nucl})$ as the kaon binding energy, where E_{nucl} is the nuclear part of the energy calculated as $\langle\Phi_0|\hat{H}_N|\Phi_0\rangle/\langle\Phi_0|\Phi_0\rangle$ with the corresponding part of the Hamiltonian,

$$\hat{H}_N \equiv \hat{T}_N + \hat{V}_{NN} - \hat{T}_{CM(N)}. \quad (9)$$

Table 1. Self consistent solution of ppK^- with various range parameters. "a" is the range parameter of $\bar{K}N$ interaction in fm. T, $V(NN)$, $V(KN,S)$, $V(KN,P)$ and $V(Coul.)$ are the total kinetic energy, the NN potential energy, the s-wave $\bar{K}N$ potential energy, the p-wave $\bar{K}N$ potential energy and the Coulomb energy, respectively. Total E. is the total energy $\langle\hat{H}\rangle$. $B(K)$ is the separate binding energy of kaon. All energies are given in MeV. R_{pp} (in fm) is the mean distance between the two protons.

	a=0.67	a=0.70	a=0.80	a=0.90
T	469.8	382.2	270.4	188.8
$V(NN)$	15.3	16.1	14.1	12.8
$V(KN,S)$	−404.8	−340.9	−255.0	−191.1
$V(KN,P)$	−130.6	−88.8	−49.3	−27.1
$V(Coul.)$	−2.2	−2.0	−1.7	−1.4
Total E.	−52.5	−33.4	−21.5	−18.0
$B(K)$	195.4	153.7	110.4	84.5
R_{pp}	1.04	1.11	1.25	1.41

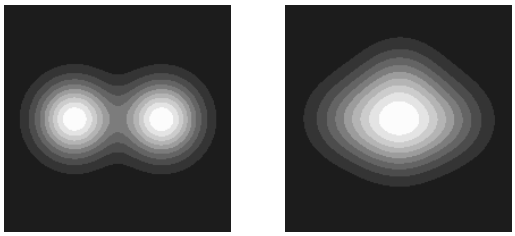
Here, $\hat{T}_N$ and $\hat{T}_{CM(N)}$ are the kinetic energy and the center-of-mass motion energy of the nucleons only.

We now show and discuss results for the ppK^- system. In the present study, the quantum numbers of ppK^- are assumed to be $J^\pi = 0^-$ and $T = 1/2$, where the parity (π) includes the intrinsic parity of the kaon. Single nucleons and the kaon are represented by two and five Gaussian wave packets, respectively. Namely, we set $N_n = 2$ in Eq. (1) and $N_K = 5$ in Eq. (2). We use a common range parameter for both s-wave and p-wave $\bar{K}N$ interactions, $a_s = a_p \equiv a$, and investigate the systematics with various range parameters a.

Fig. 3 shows the realization of the self-consistency condition for the kaon's binding energy $B(K)$, with varying range parameters. The self-consistent solution should appear on the line "Ref" which indicates $B(K)_{assumed} = B(K)_{obtained}$. As can be seen, the curves for $a \geq 0.67$ fm cross the line "Ref". This means there exists a self-consistent solution for these cases. However, for $a < 0.67$ fm, no self-consistent solution is found.

The self-consistent solutions so obtained are summarized in Table 1. As the range parameter decreases, the total binding energy (= − "Total E.") increases rapidly from 18 MeV to 53 MeV. One observes that the p-wave $\bar{K}N$ potential contributes significantly to the binding of the kaon. However, its contribution decreases as the range parameter increases. Compared to the s-wave potential, the p-wave part is still sufficiently small to justify a perturbative treatment.

In all cases, the average distance between the two protons (R_{pp}) is larger than 1.0 fm. This is attributed to the strongly repulsive core of the NN interaction. If the size of the core of a single nucleon is assumed to be roughly 0.5 fm, the present result indicates that the cores of the two protons remain well separated in space.



(a) Nucleon (b) Kaon

Fig. 4. Density contour of ppK^- calculated with the range parameter a=0.90 fm. The left (right) panel shows the distribution of the nucleons (the kaon). The size scale is 3×3 fm^2.

Fig. 4 depicts the density distribution of the nucleons and the kaon in ppK^- calculated with the range parameter $a = 0.9$ fm. The two protons keep a distance and the kaon is centered between them. In cases using other range parameters, the configuration of nucleons and kaon is essentially the same as that shown in Fig. 4.

In Table 2, we show a self-consistent solution obtained by using only the s-wave $\bar{K}N$ interaction ("Only s-wave"). Here the range parameter is 0.70 fm. We performed the same calculation as mentioned before, switching off the p-wave interaction. In this case, the total energy is similar to that obtained with the p-wave interaction ("With p-wave"). However, the decomposition of this total energy into its components is quite different. In the case "With p-wave", the ppK^- tends to increase its binding energy by the additional attraction of the p-wave potential, but simultaneously reduces binding due to the large kinetic energy, compared to the case with "Only s-wave". The proton distance (R_{pp}) of "With p-wave" is smaller than that of "Only s-wave". This is found also for other range parameters. The p-wave interaction prefers a smaller system in order to utilize larger gradients of the density distributions.

In summary, as a result of the present study of ppK^- with the AMD method using the Av18-like NN potential and the Chiral SU(3)-based $\bar{K}N$ potential, no self-consistent solutions are found when the range parameter of the $\bar{K}N$ interaction is less than 0.67 fm. The maximum total binding energy of ppK^- is 53 MeV. The configuration of the ppK^- cluster is such that the two protons keep a distance larger than 1 fm due to the strong repulsion of the short-range NN interaction. The kaon field occupies the space between them.

The present results should still be considered preliminary. The treatment of the short-range correlation is thought to be insufficient because the AMD variational wave function is based on the independent-particle picture. In the next forthcoming step a two-nucleon correlation function will be introduced in the AMD trial wave function. Since the binding mechanism of ppK^- is dominated by the strong attraction of the $\bar{K}N$ interaction, it is nonetheless expected that the essential points of the present study will not be modified substantially once the short-range correlations are adequately dealt with.

Table 2. Only s-wave interaction vs including p-wave interaction. "With p-wave" ("Only s-wave") is the result calculated with (without) the p-wave interaction for $a = 0.70$ fm.

	With p-wave	Only s-wave
T	382.2	195.3
$V(NN)$	16.1	18.9
$V(KN, S)$	−340.9	−243.1
$V(KN, P)$	−88.8	0.0
$V(Coul.)$	−2.0	−1.4
Total E.	−33.4	−30.3
$B(K)$	153.7	106.2
R_{pp}	1.11	1.36

A further important step will be the calculation of the decay width. This involves not only the mesonic mode ($\bar{K}N \to \pi Y$) but also the two-nucleon absorption process ($\bar{K}NN \to YN$). A reliable estimate of the width of antikaon-nuclear states is crucial in order to assess the experimental observability of kaonic nuclei.

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References

1. Y. Akaishi and T. Yamazaki, Phys. Rev. **C65**, (2002) 044005.
2. A. Doté, H. Horiuchi, Y. Akaishi and T. Yamazaki, Phys. Lett. **B590**, (2004) 51.
3. A. Doté, H. Horiuchi, Y. Akaishi and T. Yamazaki, Phys. Rev. **C70**, (2004) 044313.
4. T. Suzuki, Phys. Lett. **B 597**, (2004) 263; arXiv:nucl-ex/0310018.
5. M. Agnello *et al.*, Phys. Rev. Lett. **94**, (2005) 212303.
6. T. Kishimoto *et al.*, Prog. Theor. Phys. Suppl. **149**, (2003) 264; Nucl. Phys. **A754**, (2005) 383.
7. N. Herrmann, Proceedings of "International Conference on Exotic Atom and Related Topics (EXA05)" published by Austrian Academy of Science Press, pp. 73-81.
8. B. Borasoy, R. Nißler and W. Weise, Eur. Phys. J. A **25**, (2005) 79.
9. R. B. Wiringa, V. G. J. Stoks and R. Schiavilla, Phys. Rev. **C51**, (1995) 38.
10. R. Brockmann, W. Weise and L. Tauscher, Nucl. Phys. **A308**, (1978) 365; W. Weise, these Proceedings.